Question 1

Marking scheme

(a) (i) Identify correct formula
Calculate annual rate

(ii) New rate calculation
Annual interest saved
Minimum balance

(b) (i) Identify cash flows
Discount factors
Multiplication

(ii) Identify cash flows
Discount factors
Multiplication

Suggested solution

(a) (i) Effective annual rate = \((1 + r)^{\frac{n}{12}} - 1\)
Where \(r\) = rate for each time period
\(n\) = number of months in time period

\[\therefore \text{Effective annual rate} = (1 + 0.022)^{\frac{1}{12}} - 1\]
\[= 0.2984\]
\[= 29.84\%\]

(ii) The new effective annual rate = \((1 + 0.018)^{\frac{1}{12}} - 1\)
\[= 0.2387\]
\[= 23.87\%\]

The annual saving in interest per GHS outstanding = GHS\((0.2984 - 0.2387)\) = GHS\(0.0597\)

The minimum balance to be kept on the card to benefit from the change = GHS8/GHS0.0597 = GHS134.

(b) (i)

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>Discount factor</th>
<th>Present value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GHS</td>
<td>15%</td>
<td>GHS</td>
</tr>
<tr>
<td>0</td>
<td>-1,500 + 300 = -1,200</td>
<td>1.000</td>
<td>-1,200.0</td>
</tr>
<tr>
<td>1</td>
<td>500 – 150 = 350</td>
<td>0.870</td>
<td>304.5</td>
</tr>
<tr>
<td>2</td>
<td>550 – 150 = 400</td>
<td>0.756</td>
<td>302.4</td>
</tr>
<tr>
<td>3</td>
<td>500 – 150 = 350</td>
<td>0.658</td>
<td>230.3</td>
</tr>
<tr>
<td>4</td>
<td>500 – 150 = 350</td>
<td>0.572</td>
<td>200.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NPV = (-162.6)</td>
</tr>
</tbody>
</table>

(ii)

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flows</th>
<th>Discount factor</th>
<th>Present value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GHS</td>
<td>5%</td>
<td>GHS</td>
</tr>
<tr>
<td>0</td>
<td>-1,200</td>
<td>1.000</td>
<td>-1,200.00</td>
</tr>
<tr>
<td>1</td>
<td>350</td>
<td>0.952</td>
<td>333.20</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>0.907</td>
<td>362.80</td>
</tr>
<tr>
<td>3</td>
<td>350</td>
<td>0.864</td>
<td>302.40</td>
</tr>
<tr>
<td>4</td>
<td>350</td>
<td>0.823</td>
<td>288.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NPV = (86.45)</td>
</tr>
</tbody>
</table>

Final Mock Exam 1: Answers
Question 2

Marking scheme

(a) (i) Expression for total cost 2
(ii) Maximum cost 2
Batch size at maximum cost 2

(b) (i) Define variables 1
Establish constraints 2
Objective function 1
(ii) Solving equations 1
Statement of optimal solutions 2
Statement of correct simultaneous equations, 1
Solving simultaneous equations 2
Calculation of profit 3
Identifying optimal production plan 1

Suggested solution

(a) (i) The cost of packing a batch is GHS\((2 - 0.001q)q\) = GHS\((2q - 0.001q^2)\).
The delivery cost is GHS500.
The total cost \(C = -0.001q^2 + 2q + 500\).
(ii) The cost will be maximised when \(\frac{dC}{dq} = 0\) and when \(\frac{d^2C}{dq^2} < 0\).
\[
\frac{dC}{dq} = -0.002q + 2
\]
\[
\frac{d^2C}{dq^2} = -0.002 < 0
\]
\[\therefore\text{The cost is maximised when}
\]
\[0 = -0.002q + 2\]
\[\text{ie when } 0.002q = 2\]
\[q = 1,000\]
The batch size that would incur the largest possible total cost is 1,000.
The largest possible total cost = \((-0.001 \times 1,000^2) + (2 \times 1,000) + 500\) = GHS1,500.

(b) (i) Define variables
Let \(x\) = quantity of product X produced
Let \(y\) = quantity of product Y produced
Establish constraints
Material A \(2,200 \geq 0.4x + 0.3y\)
Material B \(2,500 \geq 0.2x + 0.5y\)
Non-negativity \(x \geq 0, y \geq 0\)
Construct objective function
Maximise profit \( (P) = (10 - 4.5)x + (13.50 - 7)y \)
\[ \therefore \ P = 5.5x + 6.5y \]

This assumes that fixed costs are unaffected by changes to the production plan.

(ii) There are three possible optimal solutions.

1. When materials A and B are used to the limit
2. When no units of X are produced (ie x = 0)
3. When no units of Y are produced (ie y = 0)

1. If materials A and B are used to the limit, the optimal production plan is found using the following simultaneous equations.

\[
\begin{align*}
2,200 &= 0.4x + 0.3y & (1) \\
2,500 &= 0.2x + 0.5y & (2) \\
5,000 &= 0.4x + y & (3) \\
2,800 &= 0.7y & (3) - (1) \\
4,000 &= y
\end{align*}
\]

\[ \therefore \ 5,000 = 0.4x + 4,000 \quad \text{(sub y value into (3))} \]
\[ \therefore x = 2,500 \]

**Profit**

\[ = (2,500 \times \text{GHS5.50}) + (4,000 \times \text{GHS6.50}) \]
\[ = \text{GHS39,750} \]

2. If x = 0, the material A constraint becomes

\[ 2,200 = 0.3y \Rightarrow y = 7,333 \text{ and } x = 0 \]

and the material B constraint becomes

\[ 2,500 = 0.5y \Rightarrow y = 5,000 \text{ and } x = 0 \]

Maximum value for y = 5,000 (if y = 7,333, Material B constraint is not satisfied) \( \Rightarrow \) profit

\[ = 5,000 \times \text{GHS6.50} \]
\[ = \text{GHS32,500} \]

3. If y = 0, the material A constraint becomes

\[ 2,200 = 0.4x \Rightarrow x = 5,500 \text{ and } y = 0 \]

and the material B constraint becomes

\[ 2,500 = 0.2x \Rightarrow x = 12,500 \text{ and } y = 0 \]

Maximum value for x = 5,500 (if x = 12,500, Material A constraint is not satisfied) \( \Rightarrow \) profit

\[ = 5,500 \times \text{GHS5.50} \]
\[ = \text{GHS30,250} \]

**Plan**

**Contribution**

GHS

(i) 39,750

(ii) 32,500

(iii) 30,250

\[ \therefore \ \text{Optimal production plan} \text{ is to produce 2,500 units of X and 4,000 units of Y.} \]
Question 3

Marking scheme

(a) Differentiation of functions 2
Identification of profit maximising quantity 1
Calculation of Chalk Ltd profit 2
Calculation of Cheese Ltd profit 2 7

(b) Differentiation of functions 2
Identification of profit maximising quantity 1
Calculation of Chalk Ltd profit 2
Calculation of Cheese Ltd profit 2 7

(c) Differentiation of functions 2
Identification of profit maximising quantity 1
Calculation of Chalk and Cheese Ltd profit 3 6 20

Suggested solution

(a) Chalk Ltd
\[ C = 2q^2 + 40q + 80 \]
\[
\frac{dC}{dq} = 4q + 40 = \text{marginal cost (MC)}
\]
Revenue (R) = 200q
\[
\frac{dR}{dq} = 200 = \text{marginal revenue (MR)}
\]
Profit is maximised where MC = MR: \[ 4q + 40 = 200 \]
\[ q = 40 \]
Profits at q = 40 are as follows.

<table>
<thead>
<tr>
<th></th>
<th>Chalk Ltd</th>
<th>Cheese Ltd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>8,000</td>
<td>14,400</td>
</tr>
<tr>
<td>Less costs</td>
<td>4,880</td>
<td>14,800 *</td>
</tr>
<tr>
<td>Weekly profit/(loss)</td>
<td>3,120</td>
<td>(400)</td>
</tr>
</tbody>
</table>

* \(2q^2 + 80q + 400 \) PLUS the costs of purchases from Chalk Ltd which total 200q, ie \(2q^2 + 280q + 400\).

(b) Cheese Ltd
\[ C = 2q^2 + 280q + 400 \]
\[
\frac{dC}{dq} = 4q + 280
\]
\[ R = 1,000q - 16q^2 \]
\[
\frac{dR}{dq} = 1,000 - 32q
\]
MC = MR where \[ 4q + 280 = 1,000 - 32q \]
\[ 36q = 720 \]
\[ q = 20 \]
Profits at $q = 20$ are as follows.

<table>
<thead>
<tr>
<th></th>
<th>Chalk Ltd</th>
<th>Cheese Ltd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>4,000 GHS</td>
<td>13,600 GHS</td>
</tr>
<tr>
<td>Less costs</td>
<td>1,680 GHS</td>
<td>6,800 *</td>
</tr>
<tr>
<td>Weekly profit</td>
<td>2,320 GHS</td>
<td>6,800</td>
</tr>
</tbody>
</table>

* $2q^2 + 280q + 400$

(c) Chalk and Cheese Ltd

Total costs

\[
C = 2q^2 + 40q + 80
\]

\[
\frac{dC}{dq} = 8q + 120
\]

Revenue

\[
R = 1,000q - 16q^2
\]

\[
\frac{dR}{dq} = 1,000 - 32q
\]

MR = MC when

\[
1,000 - 32q = 8q + 120
\]

\[
880 = 40q
q = 22
\]

Revenue

\[
14,256 GHS
\]

Less costs

\[
5,056 GHS
\]

Weekly profit

\[
9,200 GHS
\]
Question 4

Marking scheme

(a) (i) Negative coefficient
Explanation of constant
Explanation of gradient

(b) (i) Calculation of seasonal variation
Explanation

(ii) Calculation of forecast using regression equation
(iii) Each valid point (1.5 marks)

Suggested solution

(a) (i) We have negative correlation here, as shown by the negative coefficient of \( x \) in the regression line. That is, as the number of years employed with the company rises, so the number of days absent in a year through sickness falls.

\[
y = 15.6 - 1.2x
\]

The 15.6 represents the numbers of days absence through sickness that an employee with zero years service is expected to suffer, so it is the number of days that an employee will need off through sickness in their first year of employment. The \(-1.2\) represents the gradient of the regression line, meaning that for each extra year's service with the company, an employee will take 1.2 fewer days off sick per year.

(ii) \( y = 15.6 - (1.2 \times 8) = 15.6 - 9.6 = 6 \) days.

An employee who has been with the company for eight years is expected to require six days sick leave per year.

(iii) Limitations and problems of using this equation in practice

(1) The regression line approach presupposes that there is a linear relationship between the two variables: a sample of 50 workers has given us quite strong correlation, but still a strict linear relationship seems unlikely.

(2) A linear relationship may hold good within a small relevant range of data within which the equation may be useful in practice. But extrapolating outside this range will lead to serious inaccuracies. Thus the equation would predict that an employee with more than \( 15.6/1.2 = 13 \) years' service would have less than zero sick leave.

(3) If we use the equation to predict the future, we will use historical data to forecast the future, which is always risky.

(4) The regression line shows the expected number of days sick for a given employment period. But it is unlikely that all categories of workers will experience the same sickness pattern. The equation would be most useful if there were many employees all doing the same job in the same work conditions.
(b) (i)  

<table>
<thead>
<tr>
<th></th>
<th>Spring</th>
<th>Summer</th>
<th>Autumn</th>
<th>Winter</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>-9.8</td>
<td>-28.1</td>
<td>+11.2</td>
<td>+23.5</td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td>-7.4</td>
<td>-26.3</td>
<td>+12.5</td>
<td>+23.7</td>
<td></td>
</tr>
<tr>
<td>Average variation</td>
<td>-8.6</td>
<td>-27.2</td>
<td>+11.8</td>
<td>+23.6</td>
<td>-0.4</td>
</tr>
<tr>
<td>Adjust total variation to nil</td>
<td>+0.1</td>
<td>+0.1</td>
<td>+0.1</td>
<td>+0.1</td>
<td>+0.4</td>
</tr>
</tbody>
</table>

Estimated seasonal variation

**Seasonal variations** are short-term fluctuations in recorded values, due to different circumstances which affect results at different times of the year, on different days of the week, at different times of day, or whatever. For example, sales of ice cream will be higher in summer than in winter.

In this data, the highest output can be expected to be in the winter and the lowest in the summer.

(ii) Forecast output

\[ \text{Trend} + \text{Seasonal variation} \]
\[ = 10,536 + 23.7 \]
\[ = 10,559.7 \text{ units} \]
Question 5

Marking scheme

(a) Workings
   Tree diagram
   
   (b) (i) Workings
       Tree diagram
       
       (ii) Calculation
       
       | Marks |
       |-------|
       | 4     |
       | 4     |
       |       |
       | 8     |
       | 6     |
       | 4     |
       |       |
       | 10    |
       | 2     |
       | 2     |
       | 20    |

Suggested solution

(a)

Decision: How many to employ?

Employ 1

Employ 2

Employ 3

Expected value of profit GHS

Employ 1 person \( (0.5 \times 40,000) + (0.3 \times 50,000) + (0.2 \times 60,000) \) \( = \) GHS 47,000

Employ 2 people \( (0.5 \times 10,000) + (0.3 \times 80,000) + (0.2 \times 110,000) \) \( = \) GHS 51,000

Employ 3 people \( (0.5 \times (30,000)) + (0.3 \times 60,000) + (0.2 \times 150,000) \) \( = \) GHS 33,000

Using expected value as the basis for making a decision, the recommended decision is to employ 2 people because the EV of profit is highest.
(b) (i) Probability that the adviser will be sacked

\[
\text{Probability that the adviser will be sacked } = (0.30 \times 0.65) + (0.70 \times 0.95) \\
= 0.195 + 0.665 \\
= 0.86
\]

(ii)

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make money, sack adviser</td>
<td>0.195</td>
</tr>
<tr>
<td>Make money, do not sack adviser</td>
<td>0.105</td>
</tr>
<tr>
<td>Do not make money, sack adviser</td>
<td>0.665</td>
</tr>
<tr>
<td>Do not make money, do not sack adviser</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>

Probability that the venture capitalist made money but sacked the adviser is:

\[
0.105/(0.105 + 0.665) = 0.105/0.770 \\
= 0.1364 \text{ or } 13.64\%.
\]
Question 6

Marking scheme

<table>
<thead>
<tr>
<th>(a) Calculations for (i) to (iii) (1 mark each)</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>(b) Calculations for (i) to (iii) (2 marks each)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>(c) (i) Calculations for (1) to (4) (2 marks each)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td>(ii) Comments (1 mark per valid point)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

Suggested solution

(a) The durations of the train journeys (in hours) in order of magnitude are

1.95, 1.98, 2.00, 2.05, 2.06, 2.08, 2.09, 2.11, 2.16, 2.22

(i) \( P(> 2.10 \text{ hours}) = \frac{3}{10} = 0.3 \)

(ii) \( P(< 2 \text{ hours}) = \frac{2}{10} = 0.2 \)

(iii) \( P(\text{between 2.04 and 2.10 hours}) = \frac{4}{10} = 0.4 \)

(b) The durations follow a normal distribution with

\[ \mu = 2.07 \quad \sigma = 0.08 \]

(i) 2.10 hours is 0.03 hours above the mean.

\[ z = \frac{0.03}{0.08} = 0.375. \]

From normal distribution tables, the probability for \( z = 0.375 \) is:

\[ \frac{0.1443 + 0.1480}{2} = 0.1462 \]

Probability of a journey in excess of 2.10 hours = 0.5 – 0.1462 = **0.3538**

(ii) 2 hours is 0.07 hours below the mean

\[ z = \frac{0.07}{0.08} = 0.875 \]

From normal distribution tables, the probability for \( z = 0.875 \) is:

\[ \frac{0.3078 + 0.3106}{2} = 0.3092 \]

Probability of a journey time less than 2 hours = 0.5 – 0.3092 = **0.1908**

(iii) 2.03 hours is 0.04 hours below the mean

\[ z = \frac{0.04}{0.08} = 0.5 \]

From normal distribution tables, the probability for \( z = 0.5 \) is 0.1915

Probability of a journey time between 2.04 and 2.08 hours = 0.1915.

Probability of a journey time between 2.08 hours and 2.10 hours (see answer to (i)) = 0.1462.

Probability of journey time between 2.04 and 2.1 hours = (0.1915 + 0.1462) = **0.3377**
(c) (i) (1) \( P(\text{male reaching age of 80}) = \frac{16,199}{100,000} = 0.16199 \)

\( P(\text{female reaching age of 80}) = \frac{24,869}{100,000} = 0.24869 \)

(2) \( P(\text{male of 25 not reaching age of 50}) = \frac{85,824 - 74,794}{85,824} = \frac{11,030}{85,824} = 0.1285 \)

\( P(\text{female of 25 not reaching age of 50}) = \frac{88,133 - 78,958}{88,133} = \frac{9,175}{88,133} = 0.1041 \)

(3) \( P(\text{new born male survives until 10}) = \frac{89,023}{100,000} = 0.89023 \)

\( P(\text{new born female survives until 10}) = \frac{91,083}{100,000} = 0.91083 \)

(4) \( P(\text{male of 50 not reaching age of 80}) = \frac{74,794 - 16,199}{74,794} = \frac{58,595}{74,794} = 0.7834 \)

\( P(\text{female of 50 not reaching age of 80}) = \frac{78,958 - 24,869}{78,958} = \frac{54,089}{78,958} = 0.6850 \)

(ii) A glance at the table of figures shows that females are likely to live longer than males. The probability calculations confirm that conclusion. Such data are used by life assurance companies, which use the statistics on the large numbers of people taking out life assurance policies to determine premiums. Note that reliable statistics can only be compiled on the basis of large samples, and that life assurance companies can only use them appropriately if they write many policies. If only one person has a policy, and he or she dies early, the assurer would suffer a loss even if that person had been 'likely' to live to a great age.
Question 7

Marking scheme

(a)  
(i)  Calculating year's sales
    Analysing by market and type
    Home and overseas calculations
    Calculating proportion required

    Marks
    (i)  Calculating year's sales 1
    Analysing by market and type 2
    Home and overseas calculations 2
    Calculating proportion required 1

    (ii) Structure and presentation
        Correct bars
        Correct components

        Marks
        (ii) Structure and presentation 1
        Correct bars 2
        Correct components 2

(b)  
(i)  Calculating adjustments
    Calculating bar heights
    Histogram

    Marks
    (i)  Calculating adjustments 2
    Calculating bar heights 2
    Histogram 3

    (ii) Frequency polygon

    Marks
    (ii) Frequency polygon 2

Suggested solution

(a)  
(i)  Analysis of sales for 20X4

<table>
<thead>
<tr>
<th>Type of customer</th>
<th>Home GHSm</th>
<th>Market GHSm</th>
<th>Overseas GHSm</th>
<th>Total GHSm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household</td>
<td>128.86</td>
<td>42.70</td>
<td>171.56</td>
<td>300.98</td>
</tr>
<tr>
<td>Industrial</td>
<td>100.30</td>
<td>17.08</td>
<td>117.38</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>3.50</td>
<td>8.54</td>
<td>12.04</td>
<td></td>
</tr>
</tbody>
</table>

Household sales in the home market represent 128.86/232.66 × 100% = 55.39% of total home market sales.

Workings

1. Total sales = GHS267.3m × 1.126 = GHS300.98m
2. GHS300.98m × 0.227 = GHS68.32m
3. GHS(300.98 – 68.32)m = GHS232.66m
4. GHS300.98m × 0.57 = GHS171.56
5. GHS300.98m × 0.39 = GHS117.38
6. GHS(300.98 – 171.56 – 117.38)m = GHS12.04m
7. Household = GHS68.32m × 5/8 = GHS42.70m
   Industrial = GHS68.32m × 2/8 = GHS17.08m
   Other = GHS68.32m × 1/8 = GHS8.54m
8. Household = GHS(171.56 – 42.70)m = GHS128.86m
   Industrial = GHS(117.38 – 17.08)m = GHS100.30m
   Other = GHS(12.04 – 8.54)m = GHS3.5m
(ii) Bar chart showing analysis of sales for 20X4

Key
- Household
- Industrial
- Other

Histogram of value of orders received over a period